Network localization is unalterable by infections in bursts

Localization in networked spreading processes

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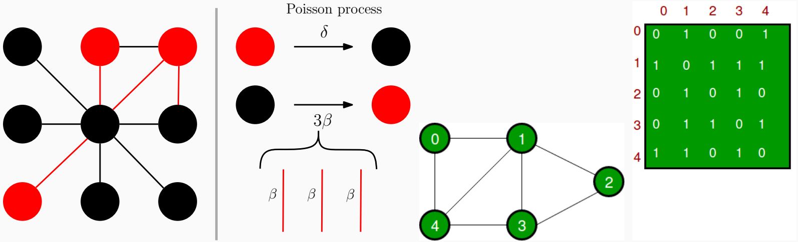
The SIS spreading model

The Susceptible-Infected-Susceptible (SIS) process on networks (*represented by the* **adjacency matrix**):

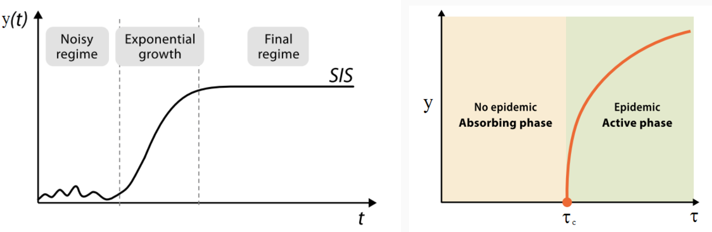
Each node is either **Infected** or **Susceptible** (healthy);

The infection and curing process are **Poisson processes** (uniform and memoriless)



The SIS model is a 2^N-state Markov process with **an absorbing all-healthy state**.

Phase transition its vanishing threshold on Scale-Free networks



Time-dependent fraction of infected nodes (prevalence) and its steady-state

Where $\tau = \frac{\text{infection rate}}{\text{curing rate}}$. Mean-field analysis:

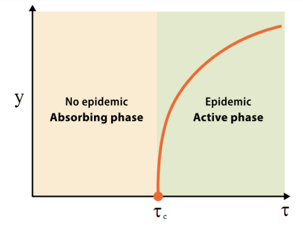
- Heterogeneous mean-field theory: $\tau_c = \frac{E[D]}{E[D^2]}$
- The N-intertwined (quenched) mean-field theory: $\tau_c = \frac{1}{\lambda_1}$

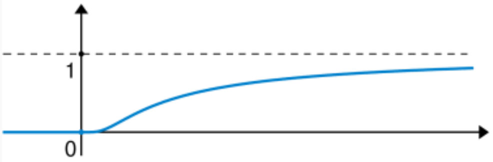
In power-law networks, $\lambda_c \to 0$ with network size.

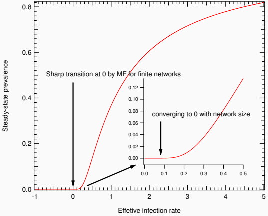
In homogeneous networks, $\lambda_c \to c > 0$.

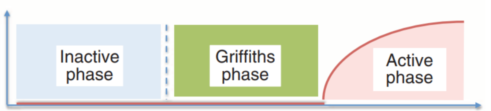
R. Pastor-Satorras and A. Vespignani, Phys. Rev. Lett. 2001; P. Van Mieghem, J. Omic and R. Kooij, IEEE/ACM Trans. on Networking (2009); Q. Liu and P. Van Mieghem, 5th Intl. Workshop CNA (2016).

Localization in the spreading process







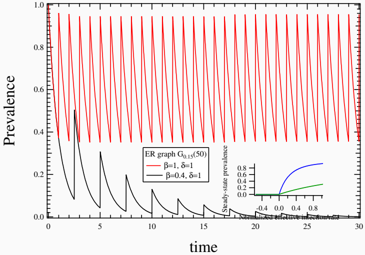


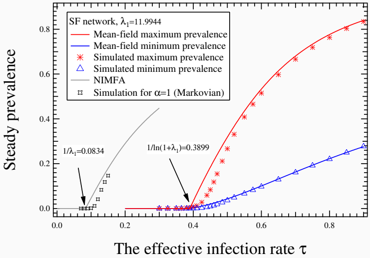
$y(\infty) = a\left(\frac{\tau}{\tau_c} - 1\right) + O\left(\frac{\tau}{\tau_c} - 1\right)$ and $a = O(N^{-c})$ for heterogenous networks.

A. V. Goltsev *et al.*, Phys. Rev. Lett., 128702 (2012); P. Moretti & M. A. Muoz, Nat. Comm. (2013)
W.y Cota, S. C. Ferreira, G. dor, Phys. Rev. E 93, 032322 (2016)

Synchronized infection with non-constant prevalence

Infection happens periodically and the period $= \frac{1}{\text{infection rate}}$.





When $\tau > \tau_c = \frac{1}{\ln(\lambda_1+1)}$, $\frac{\text{Maximum prevalence}}{\text{Minimum prevalence}} < 1 + \lambda_1$.

If $\tau \to \tau_c$, then $\frac{\text{Maximum prevalence}}{\text{Minimum prevalence}} = 1 + \lambda_1$.

Just above the threshold, $\frac{\text{Maximum prevalence}}{\text{Minimum prevalence}} = \infty$

Q. Liu and P. Van Mieghem, Phys. Rev. E 97, 022309, Feb. 2018.

Localization seems unalterable

The prevalence as a function of normalized effective infection rate $\tilde{\tau} \triangleq \tau/\tau_1 = 1$

- The maximum: $y_\infty^+(\tilde{\tau}) = a_{\max}\tilde{\tau} + o(\tilde{\tau})$
- The minimum: $y_\infty^-(\tilde{\tau}) = a_{\min}\tilde{\tau} + o(\tilde{\tau})$
- The classic SIS model: $y_\infty(\tilde{\tau}) = a\tilde{\tau} + o(\tilde{\tau})$

where

$$a_{\max} = \frac{2(\lambda_1 + 1)\ln(\lambda_1 + 1)}{\lambda_1} a = O(a \ln N) \tag{1}$$

and

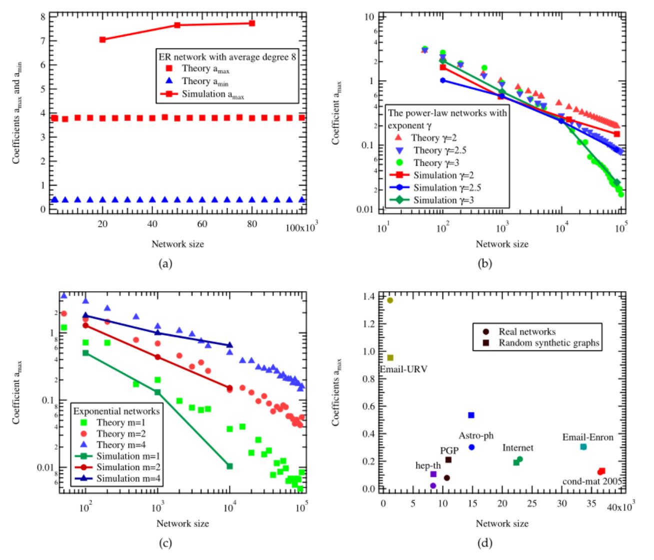
$$a_{\min} = a_{\max}/(\lambda_1 + 1)$$

Result: Since $a = O(N^{-c})$ and $a_{\max} = \frac{\ln N}{N^c}$, $a_{\max} \to 0$ with $N \to \infty$.

Structural localization of network is unalterable even for such a maximum amplification (λ_1) of spreading!

Q. Liu and P. Van Mieghem, IEEE Trans. on Netw. Sci. and Eng. (2018 arXiv: 1810.04880).

Simulation results of a_{max}



Q. Liu and P. Van Mieghem, IEEE Trans. on Netw. Sci. and Eng. (2018 arXiv: 1810.04880).