

Synchronized SIS process and a possibly largest non-Markovian threshold

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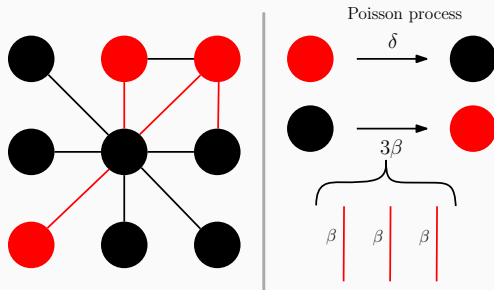
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The (Markovian) SIS process on networks

The Markovian SIS model on networks

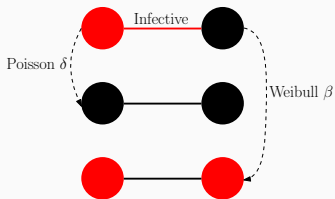
The Susceptible-Infected-Susceptible (SIS) process on networks:
Each node is either **Infected** or **Susceptible** (healthy);
The infection and curing process are **Poisson processes**
(the time length between two adjacent infections is **exponentially distributed**)

Mean-field thresholds — HMF: $\frac{\langle D \rangle}{\langle D^2 \rangle}$ NIMFA: $\frac{1}{\lambda_1}$

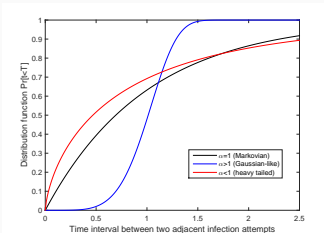
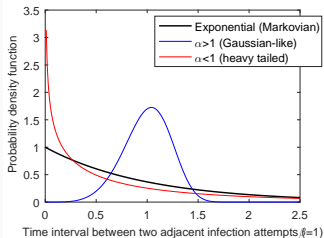


The Weibullian SIS process

SIS model with a Weibull renewal process

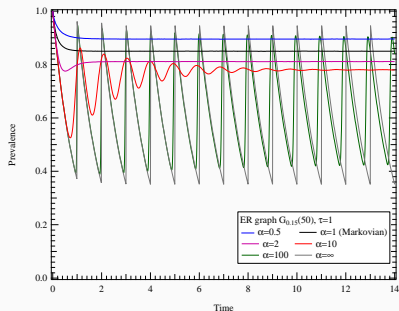


The Poisson infection process is a special case.
The infected nodes can be cured with rate δ (Poisson).



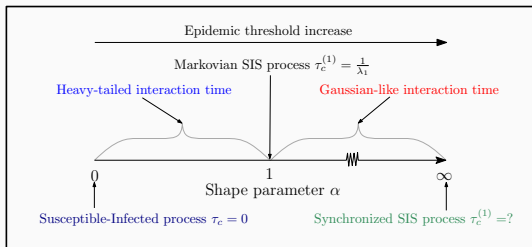
The Weibull infection process can represent very different infections: from long-tailed to Gaussian-like.

Time-dependent prevalence

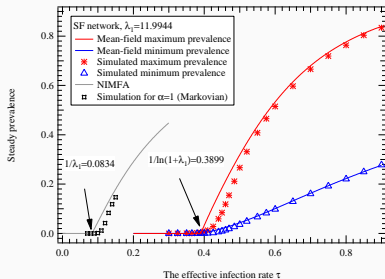
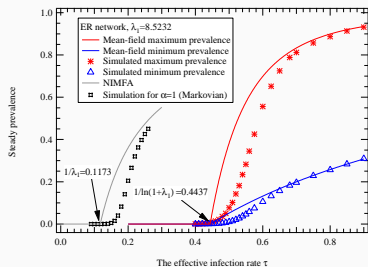


The epidemic threshold varies with α (No longer $1/\lambda_1$).

What is the range of the epidemic threshold for any infection process?



$\alpha \rightarrow \infty$ under mean-field approximation

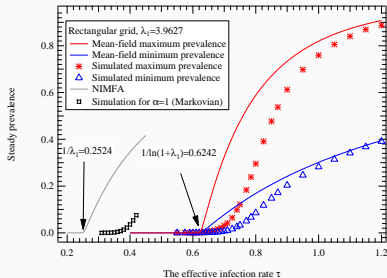


Mean-field approximation:

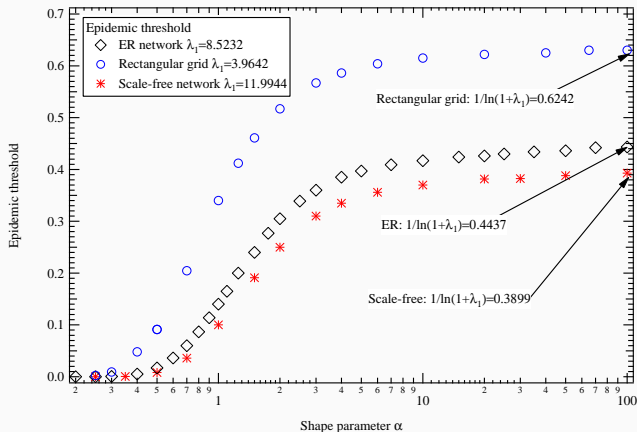
$$E[X_i(t)X_j(t)] = E[X_i(t)]E[X_j(t)].$$

The same assumption made as in NIMFA for Markovian process leads to the epidemic threshold,

$$\frac{1}{\ln(1 + \lambda_1)}$$



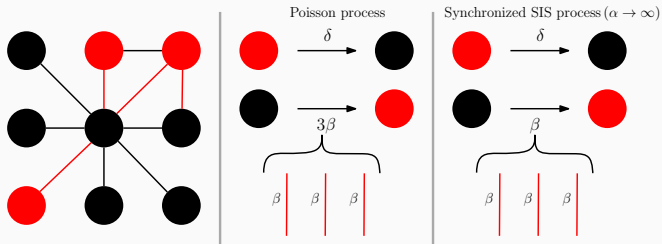
Epidemic threshold for $\alpha \in [0, \infty]$



The epidemic threshold changes approximately from 0 to $\frac{1}{\ln(1+\lambda_1)}$.

**Further discussions about the
limiting case $\alpha \rightarrow \infty$: the
synchronized SIS process**

Comparing with Markovian SIS process



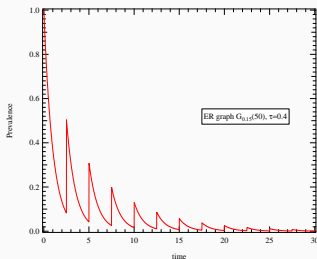
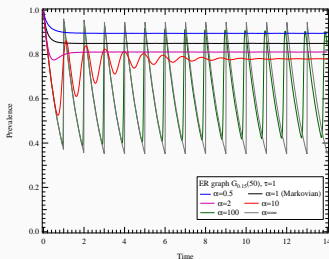
The synchronized infection ($\alpha \rightarrow \infty$) seems to be the hardest situation for the infection to persist on the network among all possible infection processes.

Multiple infected neighbors is equivalent to one.

Properties when $\alpha \rightarrow \infty$

When $\tau > \frac{1}{\ln(\lambda_1+1)}$, $\frac{\text{Maximum prevalence}}{\text{Minimum prevalence}} < 1 + \lambda_1$.

When $\tau < \frac{1}{\ln(\lambda_1+1)}$, the prevalence is upper bounded by an exponentially decreasing function of time $(e^{-\delta}(\lambda_1 + 1)^\beta)^t c$.



An example: the spreading of computer virus

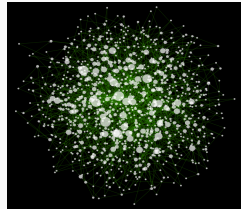
Improving the virus



Each development iteration takes $1/\beta$ time units



Burst on the network



Curing with rate δ

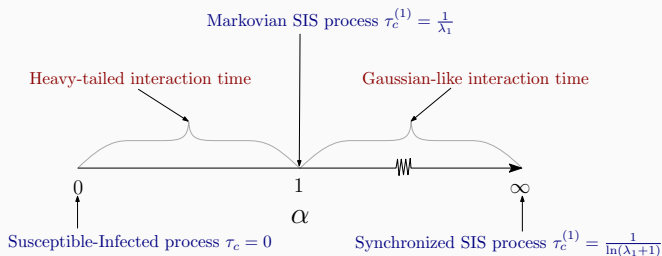
The development cycle of the virus and the topology of the network collectively determines whether the infection can persist or not.

Conclusion

We show that non-Markovian infection process can lead to a non-constant infection probability in the steady state.

We provide a possible largest epidemic threshold for the SIS process on a network for any infection process $\frac{1}{\ln(\lambda_1+1)}$.

We discussed the properties of the synchronized SIS process (the limiting case $\alpha \rightarrow \infty$).



Qiang Liu and P. Van Mieghem, Phys. Rev. E 97, 022309, (2018)
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