Synchronized SIS process and a possibly largest non-Markovian threshold

Qiang Liu
in collaboration with Piet Van Mieghem
Delft University of Technology, the Netherlands

The 6th International Conference on Complex Networks and Their Applications, Lyon, France
Nov. 29 - Dec. 01, 2017

Table of contents

The (Markovian) SIS process on networks

The Markovian SIS model on networks

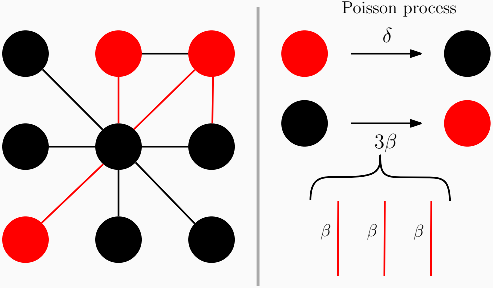
The Susceptible-Infected-Susceptible (SIS) process on networks:

Each node is either **Infected** or **Susceptible** (healthy);

The infection and curing process are **Poisson processes**

(the time length between two adjacent infections is **exponentially distributed**)

Mean-field thresholds — HMF: $\frac{<D>}{<D^2>}$ NIMFA: $\frac{1}{\lambda_1}$

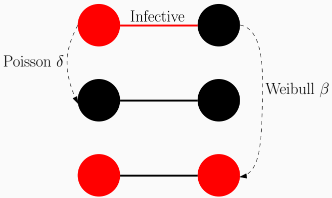


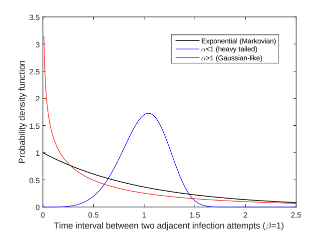
R. Pastor-Satorras and A. Vespignani, Phys. Rev. Lett. 86, 3200 (2001)

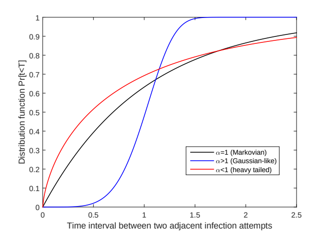
P. Van Mieghem, J. Omic and R. Kooij, IEEE/ACM Trans. on Networking, vol. 17, no. 1, Feb. 2009.

The Weibullian SIS process

SIS model with a Weibull renewal process





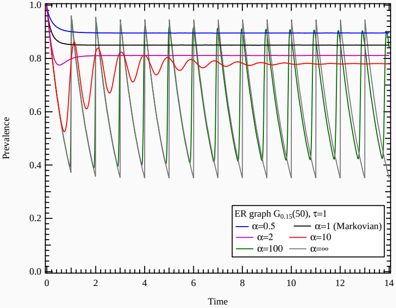


The Poisson infection process is a special case.
The infected nodes can be cured with rate δ (Poisson).

The Weibull infection process can represent very different infections: from long-tailed to Gaussian-like.

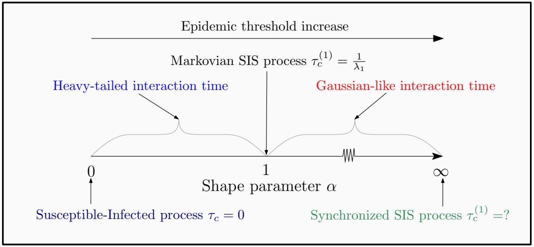
A. Vazquez, B. Racz, A. Lukacs, and A.-L. Barabasi, Phys. Rev. Lett., 98, 158702, (2007)
P. Van Mieghem and R. van de Bovenkamp, Phys. Rev. Lett., 110, 108701, (2013)

Time-dependent prevalence



The epidemic threshold varies with α (No longer $1/\lambda_1$).

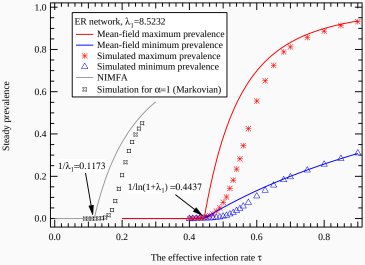
What is the range of the epidemic threshold for any infection process?

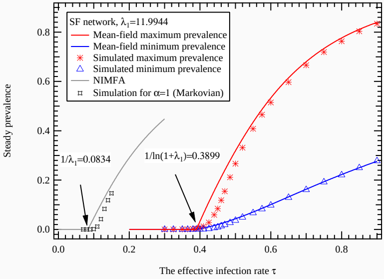


Qiang Liu and P. Van Mieghem, Phys. Rev. E 97, 022309, (2018)

P. Van Mieghem and R. van de Bovenkamp, Phys. Rev. Lett., 110, 108701, (2013)

$\alpha \to \infty$ under mean-field approximation



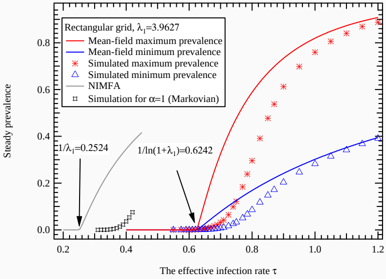


Mean-field approximation:

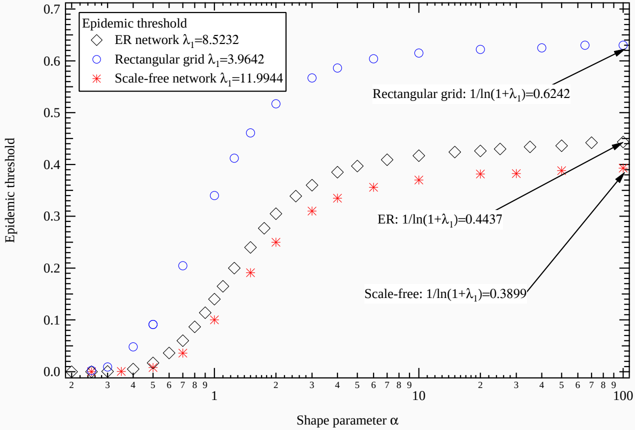
$E[X_i(t)X_j(t)] = E[X_i(t)]E[X_j(t)]$.

The same assumption made as in N-IMFA for Markovian process leads to the epidemic threshold,

$$\frac{1}{\ln(1+\lambda_1)}$$



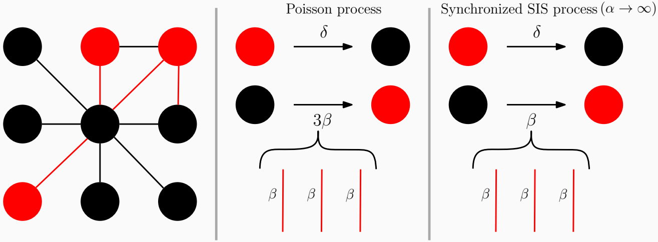
Epidemic threshold for $\alpha \in [0, \infty]$



The epidemic threshold changes approximately from 0 to $\frac{1}{\ln(1+\lambda_1)}$.

Further discussions about the limiting case $\alpha \to \infty$: the synchronized SIS process

Comparing with Markovian SIS process



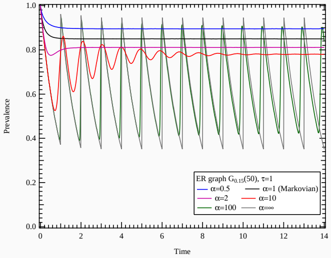
The synchronized infection ($\alpha \to \infty$) seems to be the hardest situation for the infection persist on the network among all possible infection processes.

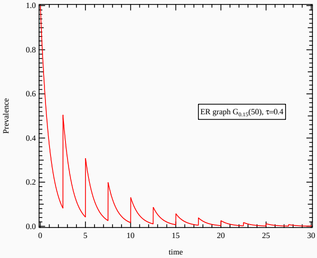
Multiple infected neighbors is equivalent to one.

Properties when $\alpha \to \infty$

When $\tau > \frac{1}{\ln(\lambda_1+1)}$, $\frac{\text{Maximum prevalence}}{\text{Minimum prevalence}} < 1 + \lambda_1$.

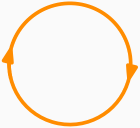
When $\tau < \frac{1}{\ln(\lambda_1+1)}$, the prevalence is upper bounded by an exponentially decreasing function of time $\left(e^{-\delta}(\lambda_1+1)^{\beta}\right)^t c$.



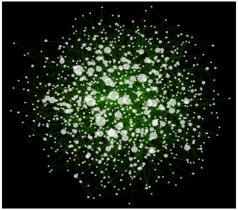


An example: the spreading of computer virus

Improving the virus



Burst on the network



Each development iteration takes $1/\beta$ time units

Curing with rate δ

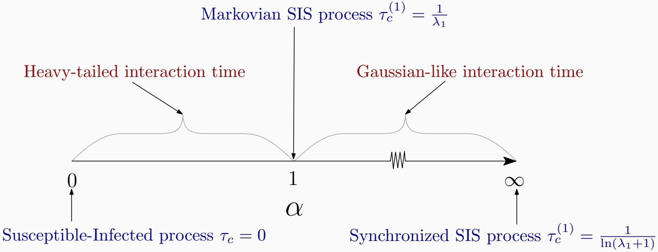
The development cycle of the virus and the topology of the network collectively determines whether the infection can persist or not.

Conclusion

We show that non-Markovian infection process can lead to a non-constant infection probability in the steady state.

We provide a possible largest epidemic threshold for the SIS process on a network for any infection process $\frac{1}{\ln(\lambda_1+1)}$.

We discussed the properties of the synchronized SIS process (the limiting case $\alpha \to \infty$).



Qiang Liu and P. Van Mieghem, Phys. Rev. E 97, 022309, (2018)
Email: Q.L.Liu@TuDelft.nl